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M2-branes wrapped on holomorphic curves

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Abstract: The generalised calibration for a wrapped membrane is gauge equivalent to the supergravity three-form under which the membrane is electrically charged. Given the relevant calibration, one can go a long way towards constructing the supergravity solution for the wrapped brane. Applications of this method have been restricted since generalised calibrations have not yet been completely classified in spacetimes with non-vanishing flux. In this paper, we take a first step towards such a classification by studying membranes wrapping holomorphic curves. Supersymmetry preservation imposes a constraint on the hermitean metric in the embedding space and it is found that this can be expressed as a restriction on possible generalised calibrations. Allowed calibrations in a particular spacetime are simply those which satisfy the constraint equation relevant to that background; in particular, we see that the previously considered Kahler calibrations are just a subclass of possible solutions.

Keywords: M-Theory, p-branes.

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Contents

1. Introduction

Solitonic M2-branes[1] are like building blocks for the BPS spectrum of M-Theory. Flat membranes preserve half of the space-time supersymmetry and a large number of supersymmetric states, preserving varying fractions of supersymmetry, can be generated either by wrapping membranes on holomorphic cycles, or by considering BPS configurations of intersecting membranes. Classical solutions corresponding to these systems exist in 11-dimensional supergravity and have been studied for a wide variety of cases [1].

We start in section 2 by reminding ourselves of some facts about membranes which will be used throughout the paper. In section 3 we introduce the notion of a calibration [2] and then extend it to define a generalised calibration [3] (see also [4]). For detailed discussions of these concepts, refer to [5].

In this paper, we consider only those static supersymmetric solutions of 11-dimensional supergravity which describe membranes wrapping holomorphic curves. Supergravity solutions for such branes can be found in several ways, of which we focus on two, in section 4. Both these methods require us to postulate a form for the metric, so in section 4.1, following Fayyazuddin and Smith we write down an ansatz which captures the isometries of the brane configuration.

[1]A parallel discussion can be carried out for M5-branes, and will appear in [11]

In section 4.2, we construct the supergravity solution by looking for bosonic backgrounds which admit Killing spinors [6]. In addition to the four-form and functions in the metric ansatz, our analysis yields a constraint on the metric in the complex subspace where the supersymmetric cycle is embedded.

In section 4.3, we adopt an alternate route [7] to solving the same problem using the technology of (generalised) calibrations. This procedure is simpler by far, but it can only be applied if we are given a generalised calibration to start with. Since there is not yet an exhaustive list to choose from, applications of this method were restricted to Kahler calibrations only. Finally, the labour of section 4.2 pays off; the metric constraint we obtained is reformulated to determine a class of generalised calibrations which exists in backgrounds with non-vanishing four form flux. This extends the class of spacetimes in which the method of [7] can be used, to find supergravity solutions.

In his talk at Strings 2000, I remember Sunil Mukhi motivating some examples with the justification that *"... You wrap the branes on everything possible, until you're blue in the face"*. Even though we may not be able to name exact shades yet, this paper attempts to at least figure out the basic colours your face might turn before you're done with wrapping membranes on holomorphic curves in $\mathbb{C}^n$!

2. Membranes: a cheat sheet

We start by collecting a few facts, both to serve as a quick reminder and to set notation for what follows. For simplicity, we consider a flat membrane with world-volume 012; $\mu = 0, 1, 2$ denotes directions along the membrane, and $\alpha = 3, 4, \ldots, 10$ spans directions transverse to the membrane.

Killing spinors: the spacetime supersymmetries preserved by the M2-brane correspond to the 16 components of the Killing spinor χ which obeys the projection $\hat{\Gamma}_{012}\chi = \chi$.

The supergravity solution: the bosonic fields in the supergravity solution are the metric and a supergravity three form which couples electrically to the membrane. For the flat membrane described above, the metric takes the form:

$$ds^2 = H^{-2/3}\eta_{\mu\nu}dX^\mu dX^\nu + H^{1/3}\delta_{\alpha\beta}dX^\alpha dX^\beta \tag{2.1}$$

and the field strength of the three form is given by:

$$F_{012\alpha} = \frac{1}{2}\frac{\partial_\alpha H}{H^2} \tag{2.2}$$

where $H = 1 + a/r^6$ is a harmonic function of the transverse radial coordinate r.

Wrapped and intersecting membranes: intersecting branes (of the same type) can often be obtained as the singular limit of branes wrapping smooth cycles. The simplest and most frequently quoted example is that of a single membrane wrapped on the holomorphic curve $f(u, v) = uv - c = 0$ in $\mathbb{C}^2$, where c is a constant. In the limiting case when $c = 0$ this curve becomes singular and can be described as a system of two orthogonal membranes which span the u and v-planes respectively and intersect at a point. In general, a complex structure can be defined on the relative transverse directions[2] and the intersecting brane configuration describes the singular limit of a membrane wrapping a holomorphic cycle in this complex subspace.[3]

We do not expect the amount of supersymmetry to vary as we change the constant in the holomorphic function, so the Killing spinors of the wrapped brane configuration ($c \neq 0$) should be the same as those for a system of n orthogonal non-overlapping membranes (the $c = 0$ limit). While we will be considering only wrapped branes, the intersecting brane limit serves as a useful check when figuring out the amount of supersymmetry preserved by the configuration. Since each additional orthogonal membrane cuts SUSY down by a factor of 2, a system of n membranes preserves $1/2^n$ of the total supersymmetry; this then, should be the amount of supersymmetry preserved by a single brane wrapping a holomorphic two-cycle in $\mathbb{C}^n$.

3. Calibrations: standard and generalised

Calibrations ϕ_p are p-forms, characteristic of a particular space-time, which enable us to classify the minimal p-dimensional submanifolds which exist in that background. A p-form ϕ_p is a standard calibration if

$$\begin{aligned} d\phi_p &= 0 \\ |\mathcal{P}(\phi_p)| &\leq |dV_{\mathcal{M}_p}|\,. \end{aligned} \tag{3.1}$$

A manifold Σ_p which saturates the above inequality is known as a calibrated manifold. For backgrounds with no flux, supersymmetry preservation requires the existence of covariantly constant spinors on the compactification manifold. These in turn imply that the manifold has reduced holonomy; calibrations on such manifolds have been classified (see for instance [9]) and include Kahler and Special lagrangian calibrations. Since in the absence of flux only minimal volume branes are stable, it follows that the volume form on a stable brane must be the pullback of a calibrated form in the ambient spacetime!

Generalised calibrations: given that calibrations emerge so naturally in the context of BPS branes, it is tempting to try and extend the concept of calibrations to include a treatment of charged p-branes. This turns out to be non-trivial; a charge gives rise to a field strength, contradicting the no-flux assumption which lead to all the earlier simplifications. Taking this into account, generalised calibrations ϕ_p are defined [3] such that

$$\begin{aligned} d(A_p + \phi_p) = 0 \qquad &\Rightarrow \qquad F_{(p+1)} = d\phi_p \\ \text{and} \qquad |\mathcal{P}(\phi_p)| &\leq |dV_{\Sigma_p}| \end{aligned} \tag{3.2}$$

hold, for any p-dimensional submanifold Σ_p.

[2]Relative transverse directions are defined to be those which are common to at least one but not all of the constituents of a system of intersecting branes

[3]The $(p-2)$ self intersection rule, (which arises from the demand that brane intersections be dynamical objects in the world-volume theory), states that BPS configurations can be constructed from a number of flat membranes if these are placed [8] such that no spatial direction is shared by any pair; a supersymmetric system of n intersecting membranes thus has a $2n$-dimensional relative transverse space.

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The most important difference between calibrations and generalised calibrations is that for the latter, the forms ϕ_p are no longer closed.[4] Notice that the invariant volume form dV_{Σ_p} carries a non-trivial contribution from the determinant of the metric on the brane.

For a p-brane wrapped on an m-cycle, $l+1$ worldvolume directions remain unwrapped ($p = l+m$). The electric potential A_{p+1} to which the brane couples, is gauge equivalent to its generalised calibration. This generalised calibration however, lives in the full spacetime and we are mostly interested in the embedding space; a generalised calibration ϕ_m can be defined on this subspace, through [3]

$$A_{p+1} = dV_{l+1} \wedge \phi_m \,, \tag{3.3}$$

where dV_{l+1} is the curved space volume form in the $(l+1)$ unwrapped directions. We will make use of this definition later, when we discuss membranes wrapped on holomorphic curves and want to focus on generalised calibrations in the complex subspace.

4. Membranes wrapped on holomorphic curves Σ in $\mathbb{C}^n$

In this section, we will make use of two different methods to find the supergravity solutions for M2-branes wrapping holomorphic curves in $\mathbb{C}^n$, for $n = 2, \ldots, 5$; $n = 2$ corresponding to the smallest complex manifold in which a two-cycle can be non-trivially embedded, and $n = 5$ being the largest complex manifold that can be contained in 11-dimensional spacetime.

As mentioned earlier, the common starting point for both these methods is an ansatz for the space-time metric, so we present this now before we go any further.

4.1 Metric ansatz

If it is to describe a static brane, all functions in the metric must be independent of time. Furthermore, for a membrane wrapped on a two-cycle in $\mathbb{C}^n$, rotational symmetry should be preserved in the $(10-2n)$ spatial directions transverse to the brane. Based on these expected isometries, the Fayyazuddin-Smith [6] ansatz for a metric describing the supergravity background created by an M2brane wrapping a holomorphic curve in $\mathbb{C}^n$ is:[5]

$$ds^2 = -H_1^2 dt^2 + 2H_1^{-1} g_{M\bar{N}} dz^M dz^{\bar{N}} + H_2^2 \delta_{\alpha\beta} dX^\alpha dX^\beta \,. \tag{4.1}$$

Here z^M are used to denote the n complex coordinates, X^α span the remaining $(10-2n)$ transverse directions and the hermitean metric in the complex space has been rescaled for latter convenience. We demand that (4.1) describes a supersymmetric configuration, and thus satisfies

$$\delta_\chi \Psi_I = \left(\partial_I + \frac{1}{4}\omega_I^{ij}\hat{\Gamma}_{ij} + \frac{1}{144}\Gamma_I{}^{JKLM}F_{JKLM} - \frac{1}{18}\Gamma^{JKL}F_{IJKL} \right)\chi = 0 \,. \tag{4.2}$$

[4]The results of the previous section are recovered, as $d\phi = 0$ when $F = 0$.

[5]This ansatz was used in [7] subject to the restriction that the metric in the complex subspace is Kahler. We are not making that assumption here

4.2 Supergravity solutions

Following the method employed in [6], we find bosonic solutions to 11-dimensional supergravity by looking for backgrounds which admit Killing spinors. Having set the gravitino to zero, we have made sure that the supersymmetric variations of the bosonic fields vanish identically and it is left only to impose that the supersymmetry variation of the gravitino vanish as well.

We require that this be true for our metric ansatz (4.1), when the variation parameter in the supersymmetry transformation is a Killing spinor. If the metric and four-form thus obtained satisfy the Bianchi Identity and the equations of motion for the field strength, they are guaranteed to obey Einstein's equations also, thereby furnishing a bosonic solution to 11-dimensional supergravity.

The first step in this process is to calculate the Killing spinors, and this is what we proceed to do now.

4.2.1 Killing spinors

A p-brane placed in a flat space-time, deforms the surrounding geometry. Supersymmetries preserved by the newly curved background can be found using the 'probe brane' approach whereby we introduce another p-brane, parallel to the one which caused the geometry to deform and calculate its Killing spinors. We call this second brane a probe because its effect on the geometry is ignored; this does not lead to any problems since the supersymmetry preserved is independent of the back-reaction of the probe[10]. The supersymmetry preserved by a p-brane with worldvolume $X^{M_0} \ldots X^{M_p}$ is given by the number of spinors which satisfy the equation

$$\chi = \frac{1}{(p+1)!}\epsilon^{\alpha_0 \cdots \alpha_p}\Gamma_{M_0 \cdots M_p}\partial_{\alpha_0}X^{M_0} \cdots \partial_{\alpha_p}X^{M_p}\chi\,, \tag{4.3}$$

where $\Gamma_{M_0 \cdots M_p}$ denotes the anti-symmetrized product of $(p+1)$ eleven-dimensional Γ matrices. The Killing spinors of a membrane wrapping a holomorphic curve Σ in $\mathbb{C}^n$ are then given by:

$$\Gamma_{0m\bar{n}}(\partial_1 X^m \partial_2 X^{\bar{n}} - \partial_2 X^m \partial_1 X^{\bar{n}})\chi = \sqrt{\det h_{ab}}\,\chi\,, \tag{4.4}$$

where

$$h_{ab} = (\partial_a X^m \partial_b X^{\bar{n}} + \partial_b X^m \partial_a X^{\bar{n}})\eta_{m\bar{n}}\,, \tag{4.5}$$

is the induced metric on Σ. Using the fact that χ is Majorana and thus of the form[6]

$$\chi = \alpha + \beta = \alpha + C\alpha^*\,, \tag{4.6}$$

we can express the constraints on the Killing spinors as:

$$\begin{aligned} \Gamma_{m\bar{n}}\alpha &= -\eta_{m\bar{n}}\alpha \\ \Gamma_{m\bar{n}}\beta &= \eta_{m\bar{n}}\beta \\ i\,\Gamma_0\alpha &= \alpha \\ i\,\Gamma_0\beta &= -\beta\,. \end{aligned} \tag{4.7}$$

[6] C denotes the charge conjugation matrix.

A membrane wrapped on a holomorphic curve in $\mathbb{C}^n$ preserves $\frac{1}{2^n}$ of the spacetime supersymmetry, corresponding to the $2^{(5-n)}$ spinors which satisfy the above conditions.

The flat space Clifford algebra written in complex coordinates[7] takes the form

$$\{\Gamma_m, \Gamma_{\bar{n}}\} = 2\eta_{m\bar{n}} \tag{4.8}$$

which resembles the algebra of fermionic creation and annihilation operators. A spinor χ in $\mathbb{C}^m$ can consequently be expressed as a sum of terms of the form $|n_1 \cdots n_m\rangle$ where n_i denotes the fermionic occupation numbers (0 or 1) corresponding to the action of the creation operator Γ_{z_i} acting on the Fock vacuum. Using this construction, the 11-dimensional spinors α and β in (4.6) can be decomposed as follows:

$$\alpha = a \otimes |0 \cdots 0\rangle \qquad \text{and} \qquad \beta = b \otimes |1 \cdots 1\rangle \tag{4.9}$$

where a and b are spinors in the $(10-2n)$ dimensional space transverse to $\mathbb{C}^n$ and due to (4.7), satisfy:

$$\begin{aligned} i\Gamma_0 a &= a \\ i\Gamma_0 b &= -b\,. \end{aligned} \tag{4.10}$$

4.2.2 The consequences of imposing $\delta_\chi \Psi_I = 0$

Having decomposed the Killing spinors as in (4.9) we can express $\delta\Psi$ as a linear combination of Fock space states, using (4.2). All these states are independent, so the coefficients of each are required to vanish seperately, giving rise to a set of relations between the metric and four form field strength.

Since the supergravity solutions for membranes wrapped on holomorphic curves in $\mathbb{C}^n$ follow a similar pattern for all n, we present the results in a unified manner, in order to avoid needless repetition.

Functions in the metric ansatz: the equations that arise from setting the gravitino variation to zero allow us to express the functions[8] in the metric ansatz (4.1) in terms of a single function H, as follows

$$\begin{aligned} \partial_I \ln H &\equiv -3\partial_I\, lnH_1 \\ &= 6\partial_I \ln H_2 \\ &= -\frac{1}{2}\partial_I \ln\left(\det g_{M\bar{N}}\right), \end{aligned} \tag{4.11}$$

where I denotes all coordinates in 11-dimensional space-time. This set of relations holds for all $\mathbb{C}^n$, with a note of caution to be sounded for $n = 5$; due to the absence of transverse directions in this case there is no H_2, but H continues to be related to H_1 and $\det g_{M\bar{N}}$ as stated above.

[7] Γ matrices for the complex coordinates are defined as $\Gamma_z = \frac{1}{2}(\Gamma_x + i\Gamma_y)$ and $\Gamma_{\bar{z}} = \frac{1}{2}(\Gamma_x - i\Gamma_y)$.

[8] As pointed out in [7], this determines H only up to a rescaling by an arbitrary holomorphic function

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The field strength: setting $\delta\Psi = 0$ also determines the four-form field strength. Non-zero components are

$$\begin{aligned} F_{0M\bar{N}\alpha} &= -\frac{i}{2}\partial_\alpha g_{M\bar{N}}\,, \\ F_{0M\bar{N}\bar{P}} &= -\frac{i}{2}[\partial_{\bar{P}} g_{M\bar{N}} - \partial_{\bar{N}} g_{M\bar{P}}]\,, \\ F_{0\bar{M}NP} &= \frac{i}{2}[\partial_P g_{N\bar{M}} - \partial_N g_{P\bar{M}}]\,. \end{aligned} \tag{4.12}$$

These expressions hold for all n, with an exception for $n = 5$; since there are no overall transverse directions, the four-form field strength can no longer have a $F_{0M\bar{N}\alpha}$ component. The only non-zero contributions in this case come from $F_{0M\bar{N}\bar{P}}$ and its complex conjugate, $F_{0\bar{M}NP}$, which are still given by expressions above.

The metric constraint: the vanishing of the gravitino variation imposes the constraint:

$$\partial[H\omega_g^{(n-1)}] = 0\,, \tag{4.13}$$

where $\omega_g \equiv ig_{M\bar{N}}dz^M dz^{\bar{N}}$ is the two-form associated with the hermitean metric.

Equation (4.13) is the central result of this paper. It enables us to characterize complex manifolds in terms of a single, simple condition on their hermitean forms. Previously, it was assumed that complex manifolds in the background of membranes wrapping holomorphic curves, were (warped) Kahler. We now see that this assumption is not necessary, except in the special case $n = 2$. There is in fact, a much larger class of complex manifolds at our disposal than we had at first suspected, corresponding to the metrics which satisfy the above constraint.

4.2.3 The Bianchi identity & equations of motion

It is clear, from the expressions (4.12), that the gauge potential is given by $A_{0M\bar{N}} = ig_{M\bar{N}}$. Hence $dF = d^2A = 0$ trivially and the Bianchi Identity is satisfied.

In order to arrive at the supergravity solution for the wrapped membranes, it is left now only to demand that the equations of motion for the field strength also be satisfied. These can be written as follows

$$\partial_I\left[\sqrt{h_{11d}}\,F^{IJKL}\right] = 0\,,$$

where h_{11d} denotes the determinant of the full eleven dimensional metric. For the wrapped membranes under consideration here, we find that $\sqrt{h_{11d}} = H^{1/3}$, regardless of the dimension of the complex subspace. The only non-trivial contribution to the equations of motion comes from

$$\partial_I\left[H^{1/3}\,F^{0M\bar{N}I}\right] = 0$$

and takes the form of a non-linear differential equation

$$\partial_\alpha^2\left[H\omega_g^{(n-1)}\right] + i2(n-1)\partial\bar{\partial}\left[H\omega_g^{(n-2)}\right] = 0\,, \tag{4.14}$$

where n denotes the dimension of the complex submanifold.

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It is only when this equation has been solved and an explicit expression for $g_{M\bar{N}}$ obtained, that the supergravity solution for the wrapped membrane can be said to be found. In practise, solving these differential equations proves to be a highly non-trivial exercise; one which is beyond the scope of this paper. We will content ourselves here with expressing all unknown quantities in terms of the hermitean metric, which can in principle be determined from (4.14). All the information needed to specify the supergravity solution is then known.

4.3 The power of calibrations

From the discussion of calibrations in section 3, we have learnt that the (pullback of the) generalised calibration corresponding to a stable wrapped brane is given simply by its volume form. Moreover, this calibration is gauge equivalent to the supergravity three form to which the membrane couples electrically; the field strength $F = dA = d\phi$ thus follows immediately. Supersymmetry requirements fix the undetermined functions in the metric ansatz in terms of a single function H, which is related to the metric through a non-linear differential equation (4.14) that follows from $d * F = 0$. We now illustrate this process[9] by applying it to the case at hand; membranes wrapping holomorphic curves.

4.3.1 A simpler method

We will now sketch the steps involved in constructing supergravity solutions for holomorphically wrapped membranes in $\mathbb{C}^n$, using generalised calibrations.

Starting with the standard metric (4.1), we can read off the gauge potential

$$A = H_1\, dt \wedge iH_1^{-1}\, g_{M\bar{N}}\, dz^M \wedge dz^{\bar{N}}\,. \tag{4.15}$$

This, in light of the above discussion, is the spacetime three-form which is (gauge equivalent to) the calibration for the membrane. Since we would like to focus only on the complex manifold and those generalised calibrations which may exist on it, we 'split up' this three-form into the product of two lower dimensional calibrations; a one-form along the time direction, and a two-form in the complex space. Comparing the gauge potential above with (3.3) we find that

$$\phi_{M\bar{N}} = iH_1^{-1} g_{M\bar{N}} \tag{4.16}$$

i.e. the generalised calibration on the complex subspace is given by the hermitean metric!

In [7], this procedure was used to construct supergravity solutions for wrapped membranes and fivebranes. However, the only calibrations considered there were Kahler. This was in no way a limitation of the approach, but merely reflected the absence of knowledge regarding other possible calibrations. As we will see in the following, the same procedure can be trivially extended to find supergravity solutions for a much larger class of calibrated branes.

[9] Details are given in the original paper [3] and in [7].

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4.3.2 Constraints

Since the calibration $\phi_{M\bar{N}}$ on the complex subspace is so intimately linked to the hermitean metric, it is obvious that by restricting the metric to obey a certain constraint, we are in fact imposing a condition on the generalised calibrations which can exist in this space.

In terms of the rescaled metric $k_{M\bar{N}} \equiv H^{1/(n-1)} g_{M\bar{N}}$ and its associated hermitean form $\omega = ik_{M\bar{N}} dz^M \wedge dz^{\bar{N}}$, the metric constraint (4.13) is

$$\partial \omega^{(n-1)} = 0\,. \tag{4.17}$$

Taking this to be the defining relation, we can look for hermitean forms which satisfy it. Each such form ω will give rise to an associated generalised calibration ϕ on the complex space, through

$$\phi = H_1^{(4-n)/(n-1)} \omega \tag{4.18}$$

It should now be clear that the Kahler calibrations considered in [7] do not exhaust the possibilities and in fact correspond only to the obvious solutions of (4.17) for which $\partial\omega = 0$.

Alternately, the constraint can be written directly in terms of calibrations. It then states that holomorphic two-form $\phi_{M\bar{N}}$ is a generalised calibration only if

$$\partial *_{\mathbb{C}} \left[\phi_{M\bar{N}} \sqrt{\det h} \right] = 0\,, \tag{4.19}$$

where $*_{\mathbb{C}}$ denotes the Hodge dual in the complex subspace and $\sqrt{\det h} = H^{(4-n)/3}$ is the square root of the determinant in the remaining, non-complex, part of space-time.

As it was subject to a non-linear differential equation, we did not have an explicit expression for the hermitean metric even when it was assumed to be Kahler; we merely expressed the four-form and the undetermined functions in our ansatz in terms of the hermitean metric. Note that these expressions remain the same, even for this wider class of solutions; only the condition on the undetermined hermitean metric is relaxed.

5. Summing up

We have seen that in the presence of a field strength, calibrated manifolds embedded in a complex subspace have a non-trivial dependence on the surrounding spacetime as well. This can be intuitively understood as follows. When the field strength is zero, Killing spinors on the complex subspace are covariantly constant with respect to the hermitean metric and the volume of a supersymmetric brane is minimized. A non-zero field strength however, generates a flux which curves the background geometry. The $\det h$ factor in (4.19) reflects the fact that the four-form flux warps the geometry of spacetime, modifying the definition of minimal (supersymmetric) cycles in the complex subspace. A new metric can be defined which incorporates the effect of the field strength into its torsion. With respect to this redefined metric, the Killing spinors of the brane configuration will be covariantly constant and, when measured by the new metric, the world-volume of a supersymmetric brane will be minimized.

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To reiterate, we have found that membranes yield stable, supersymmetric configurations when wrapped on holomorphic curves in a complex manifold for which some power[10] of the (rescaled) hermitean metric is a closed form. The volume form of the wrapped membrane is then the pull-back of a generalised calibration in 11-dimensional space-time. In particular, the class of $(1,1)$-forms which satisfies (4.19) gives a set of generalised calibrations for M2-branes wrapped on holomorphic cycles. Solutions of this condition include, but are not restricted to, Kähler calibrations.

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Note: references [12] and [13] which appeared after this paper was written also discuss the classification of BPS solutions to 11-d supergravity in backgrounds with non-vanishing flux.

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